204. Proposed by F. P. MATZ, Reading, Pa.

On a random chord in a circle two points are taken at random. What is the chance a second chord drawn at random will pass between the two points?

Solution by the late G. B. M. ZERR, Ph. D.

Let M, N be the two random points on the random chord AB; O, the center of the circle; $OA=OB=r$; $AM=y$; $MN=x$; $\angle AOB=2\theta$; μ the angle AB makes with some fixed line. Then $AB=2r\sin\theta$; an element of the circle at M is $r\sin\theta\, d\theta\, dy$; at N, $d\mu\, xdx$.

The limits of θ are 0 and $\frac{1}{2}\pi$; of μ, 0 and 2π; of y, 0 and $2r\sin\theta$; of x, 0 and y. If the second random chord intersects the first, the chance that it passes between the points is $x/2r\sin\theta$. The chance that the second chord intersects the first is $\frac{1}{3}$. Therefore, if p=the chance, we get

$$p=\frac{\frac{1}{3}\int_0^{\frac{1}{2}\pi}\int_0^{2\pi}\int_0^{2r\sin\theta}\int_0^y rx^2\sin\theta\, d\theta\, d\mu\, dy\, dx}{\int_0^{\frac{1}{2}\pi}\int_0^{2\pi}\int_0^{2r\sin\theta}\int_0^y 2r^2x\sin^2\theta\, d\theta\, d\mu\, dy\, dx}=\tfrac{1}{6}.$$

Also solved by J. E. Sanders, who obtains 3/16 as a result.

MISCELLANEOUS.

177. Proposed by R. D. CARMICHAEL, Anniston, Ala.

Sum the infinite series:

(a) $\sin x+nx\cos x-\frac{n^2x^2}{2!}\sin x-\frac{n^3x^3}{3!}\cos x+\frac{n^4x^4}{4!}\sin x+\ldots,$

(b) $\cos x-nx\sin x-\frac{n^2x^2}{2!}\cos x+\frac{n^3x^3}{3!}\sin x+\frac{n^4x^4}{4!}\cos x\ldots$

Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md., and G. B. M. ZERR, Ph. D.

(a) Arranging, we get

$$\sin x\left[1-\frac{n^2x^2}{2!}+\frac{n^4x^4}{4!}\ldots\right]+\cos x\left[nx-\frac{n^3x^3}{3!}+\frac{n^5x^5}{5!}\ldots\right]$$

$=\sin x\cos nx+\cos x\sin nx=\sin(n+1)x.$

$$\text{(b)}\quad \cos x\left[1-\frac{n^2x^2}{2!}+\frac{n^4x^4}{4!}\ldots\right]-\sin x\left[nx-\frac{n^3x^3}{3!}+\frac{n^5x^5}{5!}\ldots\right]$$

$=\cos x\cos nx-\sin x\sin nx=\cos(n+1)x.$

Also solved by Francis Rust and V. M. Spunar.